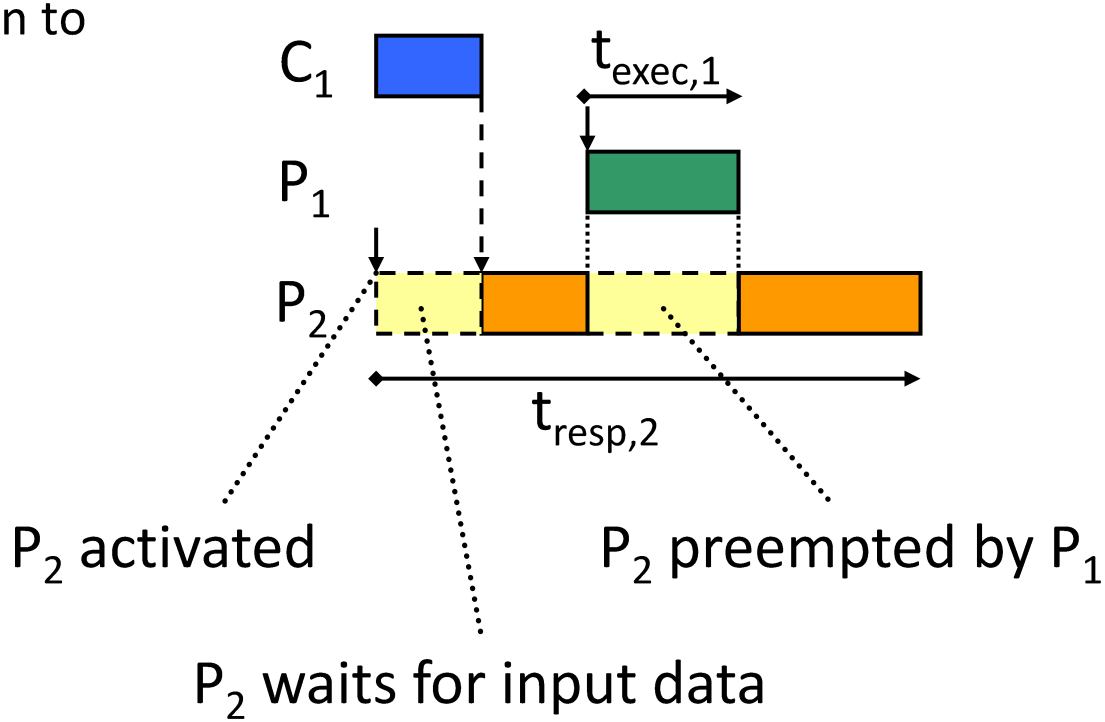
Scheduling Analysis

Part 2

Priority Driven Scheduling

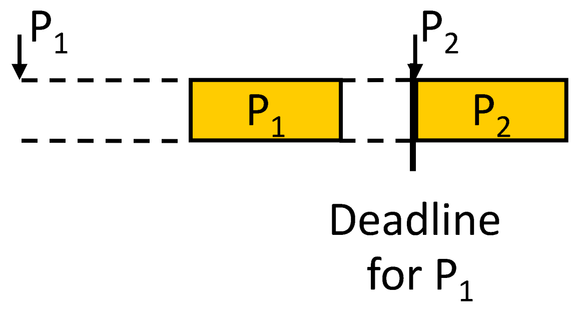
Reminder: Scheduling Analysis

- What guarantees can be made given a certain environment?
- Worst-case
  - E.g. highest resource usage/bus congestion a certain task experiences
- Response time
  - Time from task activation to end of execution
  - $t_{resp} \geq t_{exec}$
- Gantt-Charts
  - Illustrate schedules
  - Start and finish times
  - Dependencies



Rate Monotonic Scheduling (RMS)

- Priority-driven scheduling approach
- Deadlines at the end of each task's period
- Fixed priorities
  - The shorter the period, the higher the priority
- Optimal with regard to single processor scheduling
- Commonly used
- Little cost



RMS - Analysis

- Processor utilization U(n)
- A system of n independent RMS scheduled processes always meets its deadlines if (sufficient):

(1) $$\sum_{i=1}^{n} \underbrace{\frac{C_i}{T_i}}_{\text{Utilization of process i}} = U(n) \le n(2^{\frac{1}{n}} - 1)$$ (Liu/Layland ’73)

Where:
C_i: Core execution time of process i
T_i: Period of process i

- Only sufficient
  - If not met, no conclusion with respect to schedulability can be drawn

RMS – Example 1

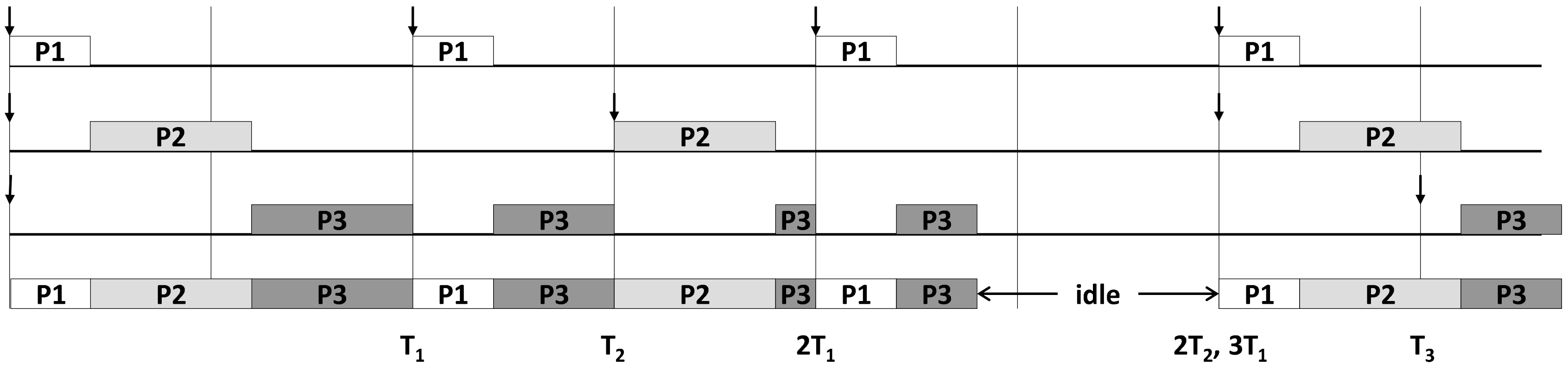
T_1=100 C_1=20

T_2=150 C_2=40

T_3=350 C_3=100

$$\sum_{i=1}^{3} \frac{C_i}{T_i} = 75{,}24\% \leq 77{,}98\% = 3(2^{1/3} - 1)$$

Equation (1) met, system schedulable



RMS – Example 2 (1/2)

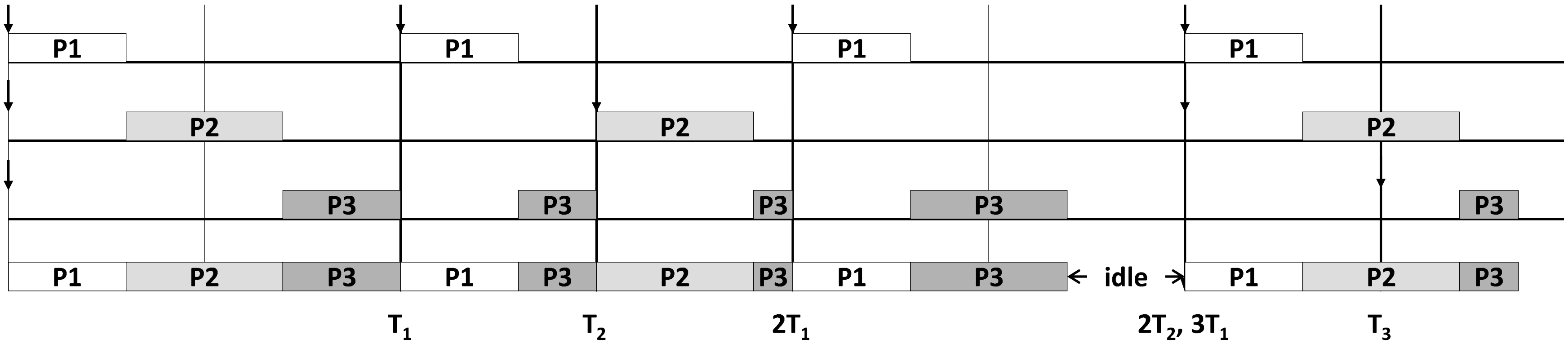
T_1=100 C_1=30

T_2=150 C_2=40

T_3=350 C_3=100

$$\sum_{i=1}^{3}\frac{C_i}{T_i} = 85{,}24\% > 77{,}98\% = 3(2^{1/3}-1)$$

Equation (1) not met, schedule not guaranteed



- How to prove that all deadlines are still met in this case?

RMS - Workload

- At each time t the accumulated requested workload is given by

$$W_n(t) = C_1\left\lceil \frac{t}{T_1} \right\rceil + C_2\left\lceil \frac{t}{T_2} \right\rceil + \ldots + C_n\left\lceil \frac{t}{T_n} \right\rceil = \sum_{i=1}^{n} C_i\left\lceil \frac{t}{T_i} \right\rceil$$

- Evaluate at $t=nT_i$ for every task i
- If the workload $W_n(t)$ at time t is less than or equal to t, sufficient resources are available

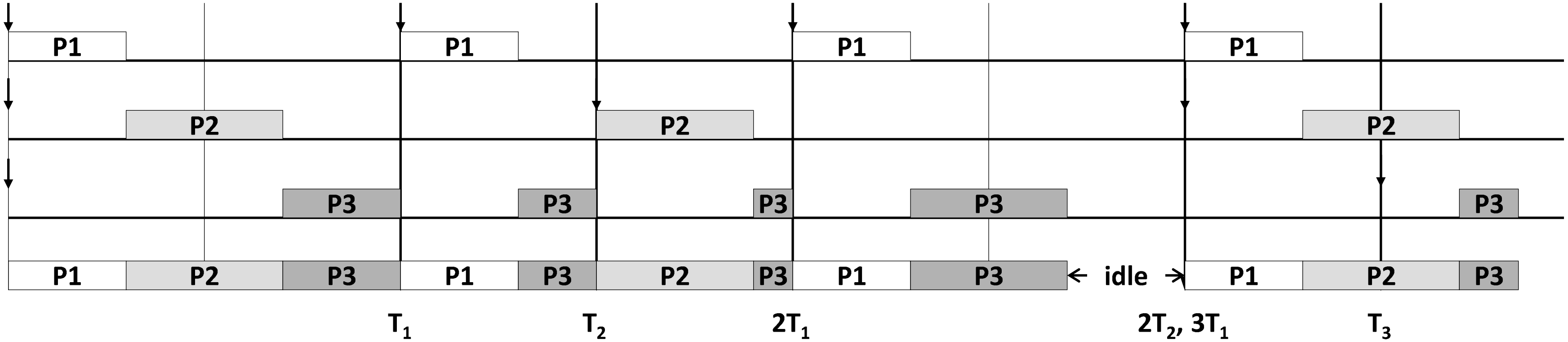
RMS – Example 2 (2/2)

T_1=100 C_1=30

T_2=150 C_2=40

T_3=350 C_3=100

$$\sum_{i=1}^{3} \frac{C_i}{T_i} = 85{,}24\% > 77{,}98\% = 3(2^{1/3} - 1)$$



Workload for n=3:

$$W_3(t) = C_1 \left\lceil \frac{t}{T_1} \right\rceil + C_2 \left\lceil \frac{t}{T_2} \right\rceil + C_3 \left\lceil \frac{t}{T_3} \right\rceil$$

(with t=350)

$$= 30 \left\lceil \frac{350}{100} \right\rceil + 40 \left\lceil \frac{350}{150} \right\rceil + 100 \left\lceil \frac{350}{350} \right\rceil = 340 \leq 350$$

RMS – Example 3

T_1=100 C_1=30

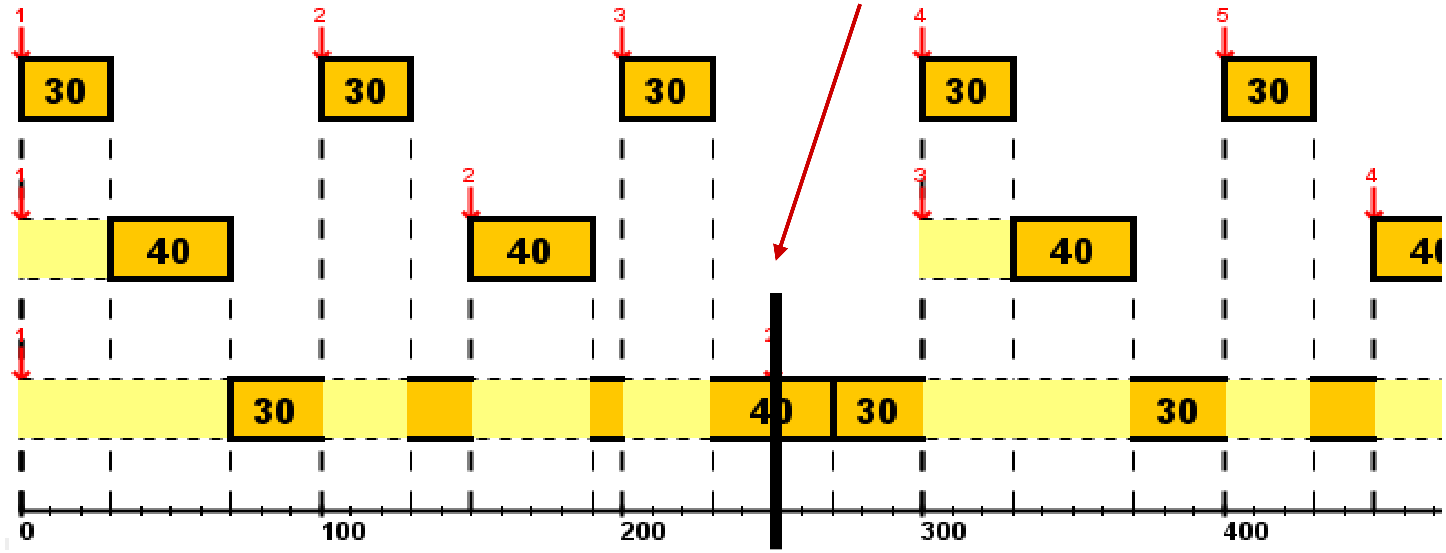
T_2=150 C_2=40

T_3=250 C_3=100

$$\sum_{i=1}^{3}\frac{C_i}{T_i} = 96{,}67\% > 77{,}98\% = 3(2^{1/3}-1)$$

Equation (1) not met, schedule not guaranteed

DEADLINE MISS!



$$W_3(t) = C_1\left\lceil\frac{t}{T_1}\right\rceil + C_2\left\lceil\frac{t}{T_2}\right\rceil + C_3\left\lceil\frac{t}{T_3}\right\rceil = 30\left\lceil\frac{250}{100}\right\rceil + 40\left\lceil\frac{250}{150}\right\rceil + 100\left\lceil\frac{250}{250}\right\rceil = 270 \geq 250$$

(with t=250)

RMS - Schedulability

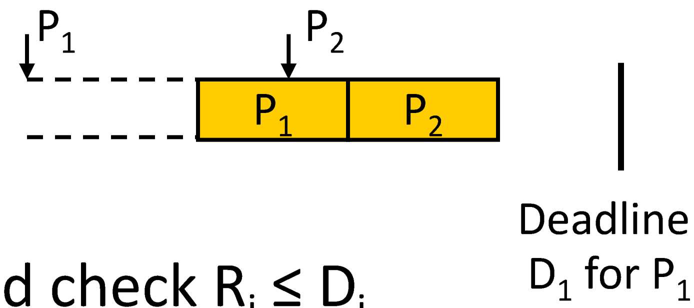
- Worst case response time has to be smaller than deadline
- How to trigger worst case behavior?
- Critical instant
  - Situation in which a certain task experiences its worst case response time
  - In case of RMS: A task is activated together with all higher priority tasks
- Given a critical instant, if a task in a RMS scheduled system meets its first deadline then it will meet all deadlines

Recursive approach:

$$R_i^n = \underbrace{C_i}_{\text{Core execution time of task i}} + \underbrace{\sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{R_i^{n-1}}{T_j} \right\rceil}_{\text{Preemption of task i by all higher priority tasks j in time window given by } R_i^{n-1}} \leq D_i, \quad R_i^0 = 0$$

Recur until fixed point R_i reached or (due to monotocity) $R_i^n > D_i$

Analysis with Arbitrary Deadlines



- Arbitrary deadlines
  - Now additional task activations during task execution/preemption possible
- Goal: Find worst case response time and check $R_i \leq D_i$
- **Windowing technique**

iterate over q=1,… {

$$w_i(q) = q \cdot C_i + \sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{w_i(q)}{T_j} \right\rceil$$

} Workload for q activations of task i (Time to finish q activations)

$$R_i(q) = w_i(q) - (q-1)T_i$$

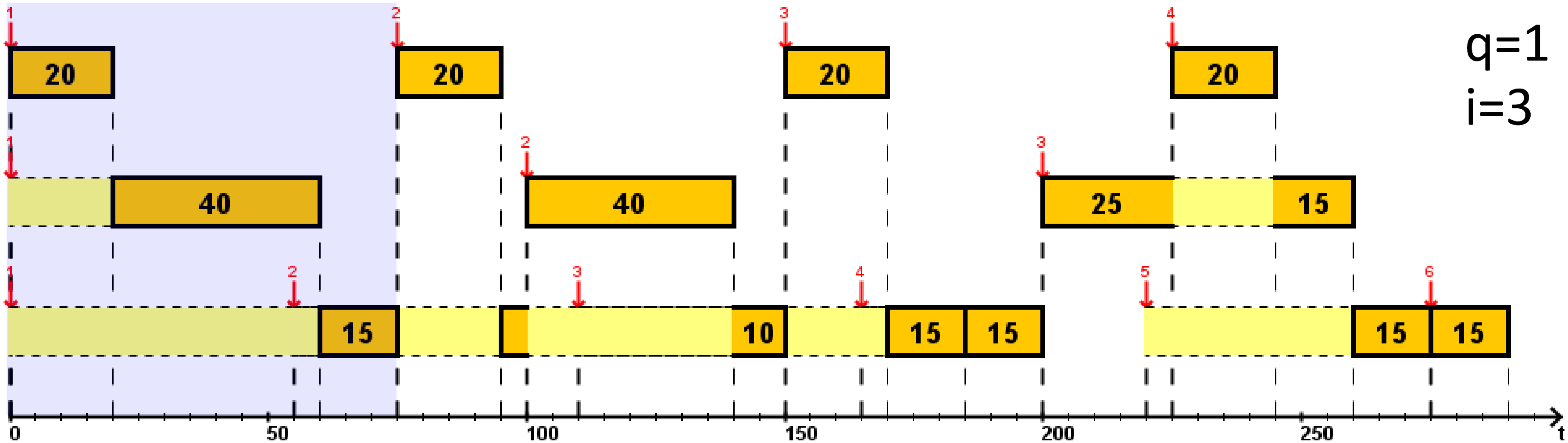
} Response time of the q'th activation of task i

} until ($w_i(q) \leq q \cdot T_i$)

} Iterate until q'th activation finishes before the q+1'th

$$R_{i,WorstCase} = \max_q \{R_i(q)\}$$

Static Priority Preemptive Scheduling (1/4)



$$w_i(1) = C_i + \sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{w_i(1)}{T_j} \right\rceil = 75$$

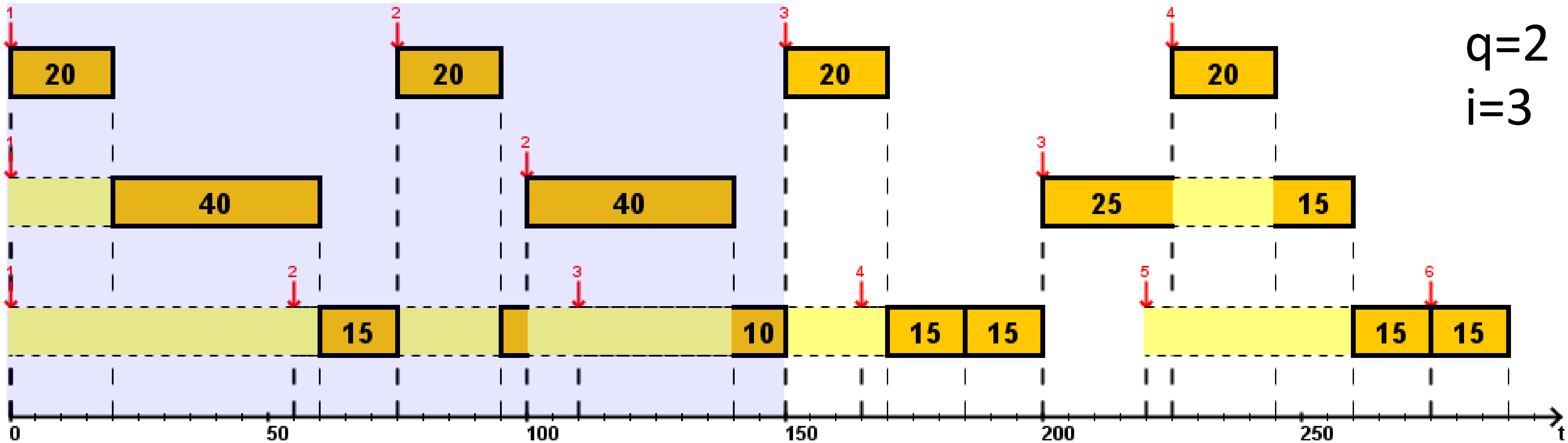
$$R_i(1) = w_i(1) - (1-1)T_i = 75 - 0 = 75$$

$w_i(1) = 75 > 55 = 1 \cdot T_i$ **→CONTINUE**

$$\begin{cases} w_3^0(1) = 0 \\ w_3^1(1) = 15 + 20\left\lceil \frac{0}{75} \right\rceil + 40\left\lceil \frac{0}{100} \right\rceil = 15 \\ w_3^2(1) = 15 + 20\left\lceil \frac{15}{75} \right\rceil + 40\left\lceil \frac{15}{100} \right\rceil = 75 \\ w_3^3(1) = 15 + 20\left\lceil \frac{75}{75} \right\rceil + 40\left\lceil \frac{75}{100} \right\rceil = 75 \end{cases}$$

Fixed point reached

Static Priority Preemptive Scheduling (2/4)

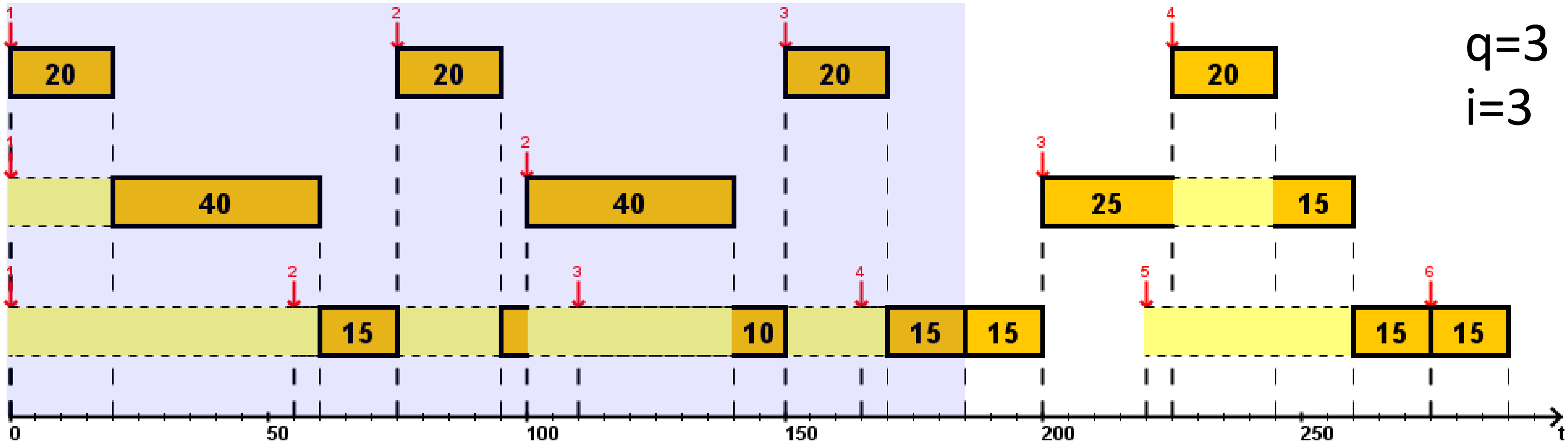


$$w_i(2) = 2C_i + \sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{w_i(2)}{T_j} \right\rceil = 150$$

$$R_i\,(2) = w_i(2) - (2-1)T_i = 150 - 55 = 95$$

$w_i(2) = 150 > 110 = 2 \cdot T_i$ →**CONTINUE**

Static Priority Preemptive Scheduling (3/4)

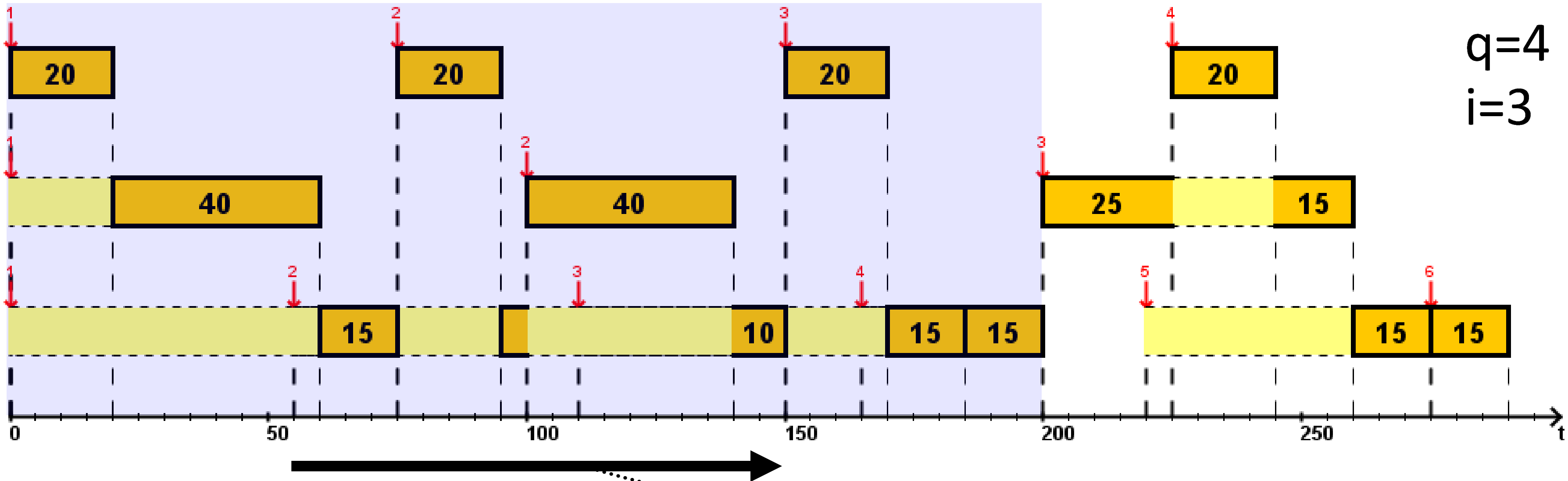


$$w_i(3) = 3C_i + \sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{w_i(3)}{T_j} \right\rceil = 185$$

$$R_i\,(3) = w_i(3) - (3-1)T_i = 185 - 2 \cdot 55 = 75$$

$w_i(3) = 185 > 165 = 3 \cdot T_i$ →**CONTINUE**

Static Priority Preemptive Scheduling (4/4)



q=4
i=3

$$w_i(4) = 4C_i + \sum_{j \in hp(i)} C_j \cdot \left\lceil \frac{w_i(4)}{T_j} \right\rceil = 200$$

$$R_i\,(4) = w_i(4) - (4-1)T_i = 200 - 3 \cdot 55 = 35$$

$$w_i(4) = 200 < 220 = 4 \cdot T_i$$ **STOP!**

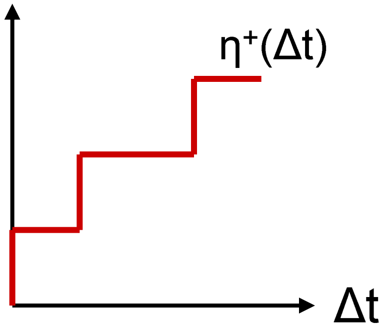
$$R_{i,WorstCase} = \max\{75, 95, 35\} = 95$$

Generalization

- Arbitrary priority assignment
- Arbitrary event models

$$R_i^n = C_i + \sum_{j=1}^{i-1} C_j \cdot \eta_j^+ \left(R_i^{n-1} \right) \leq D_i$$

$$R_i^0 = 0$$



Reminder $\eta^+(\Delta t)$:
Max. number of events that can occur in time interval Δt.

- Arbitrary event models and arbitrary deadlines

$$w_i(q) = q \cdot C_i + \sum_{j \in hp(i)} C_j \cdot \eta_j^+\left(w_i(q)\right)$$

$$R_i(q) = w_i(q) - \delta_i^-(q)$$

$$w_i(q) \le \delta_i^-(q+1)$$

$\delta^-(n)$ is the inverse of $\eta^+(\Delta t)$: Smallest interval in which any n events may occur

Concrete event models: Each P_i is known

Arbitrary event models: Only bounds ($\delta^-(q)$) for P_i are known